Uni-polar Orthogonal Codes : Design, Analysis and Applications

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Abstract— **The uni-polar orthogonal codes, with code length 'n' and code weight 'w' having auto-correlation and cross-correlation constraints equal to 1, can be designed with proposed algorithm. This proposed scheme design all the possible groups containing maximum number of orthogonal codes as per Johnson bounds. In this case, each of the designed groups is independent with other groups of uni-polar orthogonal codes having similar length, and weight. The uni-polar orthogonal codes from any of the designed groups can be utilized for generation of optical signature sequences for assignment to users of Incoherent Optical Code Division Multiple Access (OCDMA) system in spread spectrum communication over optical fiber. All other designed groups of unipolar orthogonal may be utilized for security performance improvement, increasing spectral efficiency and BER performance improvement of the OCDMA systems**

***Keywords-** auto-correlation; cross-correlation; Orthogonal binary sequences;*

I. INTRODUCTION

The uni-polar orthogonal codes are not perfect orthogonal codes because the cross correlation for uni-polar orthogonal codes can not be less than one, while the perfect orthogonal codes should have cross correlation equal to zero. Hence, the uni-polar orthogonal codes are pseudo orthogonal codes. Here uni-polar codes refer to binary sequences. A uni-polar orthogonal codeword of length 'n' and weight 'w' and its all n-1 cyclically shifted versions represent the same codeword. For the code length 'n', 2^n binary sequences can be produced, out of which binary sequences of weight 'w' can be selected. There are ${}^{n}C_{w}$ binary sequences of length 'n' and weight 'w'. These include n cyclic shifted binary sequences corresponding to each codeword. Thus total number of code words will be

$$(1/n)\ {}^{n}C_{w} = (n-1)!\ /\ w!\ (n-w)!\ ,$$
$$= (n-1)(n-2)\ldots(n-(w-1))/w.(w-1)\ldots 2.1$$

In this proposed algorithm, all the uni-polar codes of code length 'n' and code weight 'w' are generated in Difference of Positions (DoP) Representation (DoPR). From which only those codes are selected which has maximum auto-correlation for nonzero shift λ_a equal to '1'. Thereafter all possible groups having maximum number of codes are constructed such that all the codes in a group have mutual cross-correlation constraint λ_c equal to '1'. The maximum number of orthogonal codes for code length 'n', code weight 'w' auto-correlation and cross-correlation constraint equal to '1', is given by Johnson's bound A [2].

Some schemes are proposed in [3-14] for generation of optical orthogonal codes in a single group containing uni-polar orthogonal codes with auto and cross-correlation constraints lying between 1 to w-1. In these schemes the codes are generated with some extra limitations on parameters 'n', 'w', 'λ_a', 'λ_c' and 'C'. C is number of codes in a group.

The proposed scheme of generating unipolar orthogonal codes has no limitation on parameters 'n', 'w', 'λ_a', 'λ_c' except some conceptual limitations like $0 < (\lambda_a, \lambda_c) < w < n$.

This scheme generates all possible groups containing maximum number of unipolar orthogonal codes as per Johnson bound [2] while no conventional method [3-14] for designing of unipolar orthogonal codes can design more than one group of unipolar orthogonal codes. These other designed groups of unipolar orthogonal codes may be utilized for security performance improvement, increasing spectral efficiency and BER performance improvement of the OCDMA systems.

The section II describe about different representations of uni-polar orthogonal codes which is a new part of it. The section III describes the method of calculations of auto and cross-correlation constraints for unipolar orthogonal codes. In section IV, the designing method of uni-polar orthogonal codes is described as well as a new method of calculations of auto and cross correlation constraints. While section V describes about formation of groups of uni-polar orthogonal codes with auto and cross-correlation constraints equal to one. The section VI is about conclusions, applications and future scope of the research work for uni-polar orthogonal codes.

II. Representations of Unipolr Orthogonal Codes

As every uni-polar orthogonal code word also has its n cyclic shifted versions, it can have n representations which show the positions of bit 1's. This type of representation of a uni-polar orthogonal codeword may be called as weighted positions representation (WPR) or bit 1's positions representation. These n representations can be reduced by making a compulsory position of bit 1 at position zero. This reduce the number of weighted positions representations of the orthogonal code to w. This reduced weighted positions' representation may be called as compulsory weighted positions' representation or fixed weighted positions representation (FWPR). Such representation of an orthogonal code is not unique; it has w representations of an orthogonal code. To make the representation of an orthogonal code to be unique, Difference of Positions of bit 1's is proposed. An orthogonal codeword represented in weighted positions' representation or fixed weighted positions' representation has fixed w difference of positions (DoP) of bit 1's in serial and cyclic order. The DoP elements can be calculated by using modulo n addition/subtraction. There are w DoP representations of an orthogonal code word. Each of DoP representation has w DoP elements. If the lowest or highest DoP element is fixed as first or last element respectively, the orthogonal code can be represented in unique manner called fixed lowest DOP or fixed highest DOP representation. It can be best understood by the following example.

Example: code length n = 13, weight w = 3

No. of orthogonal code words = 12.11 / 3.2.1 = 22

A code from above example of n =13, w =3 is Code = 1010001000000. It can be represented in weighted positions representation (WPR) as (0, 2, 6). This code has 13 weighted positions representations due to its cyclic shifted versions viz. (1,3,7), (2,4,8), (3,5,9), (4,6,10), (5,7,11), (6,8,12), (7,9,0) or (0,7,9), (1,8,10), (2,9,11), (3,10,12), (4,11,0) or (0,4,11), (1,5,12), (2,6,0) or (0,2,6). These representations of an orthogonal code can be reduced by making a fixed position of bit '1' at zero like (0,2,6), (0,4,11), (0,7,9). There are w=3 fixed weighted positions representations.

All of the weighted positions representations (WPR) or fixed weighted positions representations can be changed into difference of positions of bit 1's representations in the given following manner, the code representation (0,2,6) is equivalent to (2-0, 6-2, 0-6)mod(13) = (2,4,7) in DoP representation.

Similarly WPR (0, 4, 11) =DoP (4, 7, 2),

WPR (0, 7, 9) = DoP (7, 2, 4),

WPR (2, 4, 8) = DoP (2, 4, 7), and

WPR (2, 9, 11) = DoP (7, 2, 4),

Similarly for other WPRs there are only 3 DoP representations viz. DoP (2, 4, 7) or DoP (4, 7, 2) or DoP (7, 2, 4). These DoP representations of an code are cyclic shifted versions of each others.

These w DoPs can be fixed to lowest element at first position as DoP (2,4,7) or highest element fixed at last position as DoP (2,4,7), here in this case both representations are same but these may be different also in other cases. Hence, unique representation of uni-polar orthogonal code = 1010001000000 is DoP (2, 4, 7). The unique representation of DoP (a, b, c, d) of weight w =4 can be converted into WPR as

DoP (a, b, c, d) = WPR (0, a, a+b, a+b+c).

III. Calculation of Correlation Constraints

An uni-polar orthogonal code is represented by n binary sequences for every cycle shifting of the code in WPRs. The correlation of an uni-polar orthogonal code with its un-shifted binary sequence is equal to weight 'w' of the code

Suppose code X with code length 'n' and weight 'w' be $X = (x_0\ x_1\ x_2\ .\ .\ .\ x_{n-1})$, $x_t = 0$ or 1 for $0 \le t \le n-1$

The correlation of X with its un-shifted sequence is given by

$$R_{XX} = \sum_{t=0}^{n-1} x_t x_t$$

which will be always equal to w. It is also auto-correlation peak which appear at the detector for the detection of binary data equal to '1' represented by this codeword.

The code X with m unit cyclic left shifting is represented as

$X_m = (x_m\ x_{m+1}\ x_{m+2}\ .\ .\ .\ x_{m-1})$, x_{m+t} is given under modulo n addition for $0 \le m \le n-1$,

The correlation of X with X_m (the cyclically shifted versions) is given by

$$R_{XX_m} = \sum_{t=0}^{n-1} x_t x_{t\oplus m} \quad 0 < m \le n-1$$

The auto-correlation constraint λ_a is defined and given as

λ_a = Maximum of $(R_{XX_1}, R_{XX_2}, ..., R_{XX_{n-1}})$ or

$$\lambda_a \ge \sum_{t=0}^{n-1} x_t x_{t\oplus m}.\ 0 < m \le n-1$$

For uni-polar orthogonal binary sequences,

$$0 \le \lambda_a \le w-1$$

Suppose code Y with code length 'n' and weight 'w' be

$Y = (y_0\ y_1\ y_2\ .\ .\ .\ y_{n-1})$, $y_t = 0$ or 1 for $0 \le t \le n-1$

The correlation of X with Y and its circularly un-shifted & shifted binary sequences (Y_m) is given as

$$R_{XY_m} = \sum_{t=0}^{n-1} x_t y_{t\oplus m} \quad 0 \le m \le n-1$$

The cross-correlation constraint λ_c is defined and given as λ_c = Maximum of $(R_{XY_0}, R_{XY_1}, ..., R_{XY_{n-1}})$ or

$$\lambda_c \ge \sum_{t=0}^{n-1} x_t y_{t\oplus m} \quad 0 \le m \le n-1$$

For uni-polar orthogonal binary sequences

$0 \le \lambda_c \le w-1$ [1].

IV. Design of Uni-polar Orthogonal codes

As in the above section for representations of uni-polar orthogonal codes, it is found that an uni-polar orthogonal binary sequences of length 'n' and weight 'w' can be uniquely represented in DoP representation by fixing of first lowest element or last highest element. Suppose we are taking the case of first lowest, then all possible codes of length 'n' and weight 'w' can be generated by writing a MATLAB based program. It can be understand by following example,

Code length n =13, weight w =3, in DoP representation,

Number of DoP elements equal to weight w i.e. 3,

Sum of all DoP elements is equal to code length n i.e. 13

Suppose code is (a, b, c = n – (a+b)) in DoPR,

First method for designing of UOC can be described as following

Codes can be generated with following conditions for w=3.

(ι) $a > 0, b > 0, c > 0,$

(ιι) $a \le b,$

(ιιι) $a < c,$

The codes are generated as following

Code #1	(a=1, b=1, 11)
Code #2	(a=1, b=2, 10)
Code #3	(a=1, b=3, 9)
…	(…, …, …)
Code #21	(a=3, b=6, 4)
Code #22	(a=4, b=4, 5)

Number of codes generated is (n-1)(n-2)/w.(w-1)…1 = 12.11/3.2.1 = 22, hence number of codes are verified.

For weight w > 3, w elements are initialized and varied such that all w elements are positive. First element is always less than last element but when less than or equal to other elements, then first element is less than or equal to second, second element is less than or equal to third and so on upto last element.

Second method for designing of UOC can be described as follows, in DoPR with last highest element, all codes can be generated for same above example, n=13, w=3

Number of DoP elements equal to weight w i.e. 3,

Sum of all DoP elements is equal to code length n i.e. 13

Suppose code is (a, b, c = n – (a+b)) in DoPR,

Codes can be generated with following conditions for w=3.

(ι) $a > 0, b > 0, c > 0,$

(ιι) $b \le c,$

(ιιι) $a < c,$

The codes are generated as following

Code #1	(a=1, b=1, 11)
Code #2	(a=1, b=2, 10)
Code #3	(a=1, b=3, 9)
…	(…, …, …)
Code #21	(a=5, b=1, 7)
Code #22	(a=5, b=2, 6)

For weight w > 3, w elements are initialized and varied such that all w elements are positive. The last element is always greater than first element but when greater than or equal to other elements, then first element is less than or equal to second, second element is less than or equal to third and so on upto last element.

All possible combination of codes can be generated for code length 'n' and weight 'w', by writing a MATLAB based program for it.

Now next challenge is to select only those codes which have λ_a and λ_c equal to '1'. Then first task is to find the value of λ_a for these all possible codes for code length 'n' and weight 'w'. Actually λ_a is given as, for an orthogonal code X

$$\lambda_a \ge \sum_{t=0}^{n-1} x_t x_{t\oplus m}, \quad 0 < m \le n-1.$$

In binary digit representation

$X = (x_0\ x_1\ x_2\ .\ .\ .\ x_{n-1})$, $x_t = 0$ or 1 for $0 \le t \le n-1$

$X_m = (x_m\ x_{m+1}\ x_{m+2}\ .\ .\ .\ x_{m-1})$,

x_{m+t} is given under modulo n addition for $0 < m \le n-1$,

The orthogonal code remains same for each shift, i.e.

$X = X_1 = X_2 = X_3 = . . . = X_n$

The code X in FWPRs can be given as

$X_{F0} = X_0(f_0, f_1, f_2, …, f_{w-1})$ means that the position $X_0(f_0)$,

$X_0(f_1)$, $X_0(f_2)$, … , $X_0(f_{w-1})$, are '1' (weighted) while other positions are '0'.

Now shifting X_{F0} by $X_0(f_1)$, $X_0(f_2)$, …, $X_0(f_{(w-1)})$, units in left circularly to get X_{F1}, X_{F2}, …$X_{F(w-1)}$.

$X_{F1} = X_1(f_0, f_1, …, f_{w-1})$

$X_{F2} = X_2(f_0, f_1, …, f_{w-1})$

…

$X_{F(w-1)} = X_{(w-1)}(f_0, f_1, …, f_{w-1})$

The correlation of X_{F0} with X_{F1}, X_{F2}, . . ., $X_{F(w-1)}$ can be calculated as $R_{X_{F0}X_{F1}} = X_{F0}X'_{F1} = \sum_{t=0}^{w-1} X_0(f_t)X_1(f_t)$.

$$R_{X_{F0}X_{F2}} = X_{F0}X'_{F2} = \sum_{t=0}^{w-1} X_0(f_t)X_2(f_t)$$

…

$$R_{X_{F0}X_{F(w-1)}} = X_{F0}X'_{F(w-1)} = \sum_{t=0}^{w-1} X_0(f_t)X_{(w-1)}(f_t)$$

Here X'_F represent the transposed matrix of X_F and

$$X_i(f_s).X_j(f_t) = \begin{cases} 1, & for \quad X_i(f_s) = X_j(f_t) \\ 0, & otherwise \end{cases}$$

$$0 \le i, j, s, t \le w-1$$

λ_a = Max ($R_{X_{F0}X_{F1}}$, $R_{X_{F0}X_{F2}}$, …, $R_{X_{F0}X_{F(w-1)}}$)

or $\lambda_a \ge \sum_{t=0}^{w-1} X_0(f_t)X_i(f_t). 1 \le i \le w-1$

Similarly for two codes X and Y of same code length 'n' and weight 'w', the cross-correlation constraint λ_c can be calculated in FWPR,

$X_{F0} = X_0(f_0, f_1, f_2, …f_{w-1})$

$Y_{F0} = Y_0(f_0, f_1, f_2, …f_{w-1})$

Now shifting Y_{F0} by $Y_0(f_1)$, $Y_0(f_2)$, …, $Y_0(f_{(w-1)})$, units in left circularly to get Y_{F1}, Y_{F2}, …$Y_{F(w-1)}$.

$Y_{F1} = Y_1(f_0, f_1, f_2, …f_{w-1})$

$Y_{F2} = Y_2(f_0, f_1, f_2, …f_{w-1})$

…

$Y_{F(w-1)} = Y_{(w-1)}(f_0, f_1, f_2, …f_{w-1})$

The correlation of X_{F0} with Y_{F0}, Y_{F1}, Y_{F2}, . . ., $Y_{F(w-1)}$ can be calculated as $R_{X_{F0}Y_{F0}} = X_{F0}Y'_{F0} = \sum_{t=0}^{w-1} X_0(f_t)Y_0(f_t)$.

$$R_{X_{F0}Y_{F1}} = X_{F0}Y'_{F1} = \sum_{t=0}^{w-1} X_0(f_t)Y_1(f_t)$$

$$R_{X_{F0}Y_{f2}} = X_{F0}Y'_{F2} = \sum_{t=0}^{w-1} X_0(f_t)Y_2(f_t)$$

…

$$R_{X_{F0}Y_{F(w-1)}} = X_{F0}Y'_{F(w-1)} = \sum_{t=0}^{w-1} X_0(f_t)Y_{(w-1)}(f_t)$$

$$X_i(f_s).Y_j(f_t) = \begin{cases} 1, & for \quad X_i(f_s) = Y_j(f_t) \\ 0, & otherwise \end{cases}$$

$$0 \le i, j, s, t \le w-1$$

$$\lambda_c = \text{Max}\ (R_{X_{F0}Y_{F0}}, R_{X_{F0}Y_{F1}}, R_{X_{F0}Y_{F2}}, ..., R_{X_{F0}Y_{F(w-1)}})$$

$$\lambda_c \ge \sum_{t=0}^{w-1} X_0(f_t)Y_i(f_t),\ 0 \le i \le w-1$$

Suppose the code is represented in DoPR with first lowest element or last highest element, the codes in DoPR can be converted into FWPR to find λ_a and λ_c as follows

The code X in DoPR is given as

$X_{D0} = X_0(d_0, d_1, . . . d_{w-1})$, its equivalent in FWPR is given as

$X_{F0} = X_0(f_0, f_1, . . . f_{w-1})$, where

$X_0(f_0) = 0$,

$X_0(f_1) = X_0(d_0)$,

$X_0(f_2) = X_0(d_1)$,

…

$X_0(f_{w-1}) = X_0(d_{w-2})$

Once X_{F0} is formed from X_{D0},

X_{F0} is shifted by $X_0(f_1)$, $X_0(f_2)$, …, $X_0(f_{(w-1)})$, units in left circularly to get X_{F1}, X_{F2}, …$X_{F(w-1)}$ respectively.

Similarly Y_F can be generated from Y_{D0}, the code Y in DoPR. Once the X_F and Y_F is generated then λ_a

and λ_c can be calculated for any pair of codes as described above.

V. FORMATION OF GROUPS OF UNIPOLAR ORTHOGONAL CODES

In above section all possible uni-polar orthogonal codes can be generated. For each of these code the auto-correlation constraint λ_a can be calculated. Now a big group of those uni-polar orthogonal codes can be formed in which each code has $\lambda_a = 1$. From this big group only those codes have to be selected which have $\lambda_c = 1$ with each other.

Suppose there are 'A' codes which have $\lambda_a = 1$, the correlation matrix AxA can be formed which contain all the value of λ_a and λ_c of these A codes with each other. This correlation matrix can help in selection of those codes which have $\lambda_c = 1$

The Johnson's bound for code length 'n', weight 'w', and

$\lambda_a = \lambda_c = 1$ is given as (n-1)/w(w-1) = J_A. This J_A gives the maximum number of orthogonal codes in the group for which λ_a and $\lambda_c = 1$. The J_A codes from A number of codes can be grouped in G = ${}^{A}C_{J_A}$ manner or different groups. The J_A codes in each of G groups are checked for $\lambda_c = 1$ with each other from correlation matrix AxA. From these G groups H such groups are selected which has $\lambda_c = 1$. These independent each of H groups contain only those codes which have λ_a and $\lambda_c = 1$. The J_A uni-polar orthogonal codes of any of H selected groups can be used as optical signature sequences by putting an optical pulse at weighted positions in incoherent optical cdma system.

VI. CONCLUSION

The proposed scheme for the generation of uni-polar orthogonal codes is able to generate not only one set of orthogonal codes but all other possible sets of uni-polar orthogonal codes with auto and cross-correlation constraints equal to one, for same code length 'n' and weight 'w'. In future the work can be extended for the generation of groups of two dimensional uni-polar orthogonal codes of desired or given value of auto and cross-correlation constraints. The main application of this scheme is in the generation of uni-polar one-dimensional optical orthogonal signature sequences to be utilized in incoherent optical CDMA system. The different generated possible sets of orthogonal codes can be utilized for security purpose as well as for increasing spectral efficiency performances and BER performance improvements of optical code division multiple access systems.

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